Robust Factor-Based Investing

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Markowitz [1952] introduced a framework based on portfolio return and risk, often referred to as the *mean–variance framework*, for developing numerous contributions to portfolio optimization and investment management. The three most notable drawbacks of mean–variance analysis are the difficulty in obtaining accurate input estimates, the sensitivity of the optimal portfolio to its inputs, and the limitation of using only one factor to drive returns (i.e., the market factor). Limiting the factors' impact on returns to one factor and combining optimization with estimation are concerns for asset managers. Robust factor-based models address these three concerns by performing asset estimation based on factors and enhancing portfolio stability through robust models.

Factor models are now an essential part of asset management for analyzing the risk characteristics of an individual security and portfolio, managing a portfolio's exposure to the sources of portfolio risk, and reducing errors in estimating individual asset returns. Moreover, the identification of major factors in different markets led to investment strategies known as *factor investing* (Ang [2014]). These factor-based methods control portfolio exposure in specific factors that reflect the desired views of asset managers. Recently, rule-based strategies packaged as smart beta investments have become popular factor-based approaches. As interest in factor-based investing expands, robust factor models are gaining more attention for addressing the uncertainty in factor-based investments.

Factors affect movements in financial markets; thus, identifying important factors and analyzing the relationship between factors and the overall market enable portfolio managers to understand the source of market movements and make investment decisions accordingly. The development of factor-based robust models seems inevitable because estimation errors and model errors will be better controlled if managers concentrate on the main market characteristics when building robust models.

In this article, we provide a survey of studies on robust methods for factor-based investing. Although not limited to robust portfolio optimization strategies, many studies on robust factor-based methods apply the worst-case approach of robust optimization in which uncertainty in asset returns is expressed using a factor model. Depending on how the robust formulations are constructed, the strategies can be applied to investment in asset classes, investment in individual stocks, investment in factor indices, or investment for tracking an index.

We begin by briefly introducing robust portfolio models before discussing the use of robust factor models for portfolio management. In addition to reviewing factor-based strategies and factor exposure of robust portfolio optimization, we comment on how robust optimization is utilized in practice for factor-based investing.[1]

ROBUST PORTFOLIO MANAGEMENT

Many portfolio strategies have their roots in the mean–variance model in which portfolio return and risk are estimated by mean and variance, respectively. Although higher portfolio returns are associated with higher portfolio risk, the goal is to consider the tradeoff between risk and return to reach optimal portfolios for a given investment situation. However, the most notable obstacle in adopting the mean–variance framework in practice is accurately estimating the required inputs, such as the vector of mean asset returns and the covariance matrix, which includes variances and covariances of asset returns.

Another main issue with mean–variance portfolios is the high sensitivity of portfolio weights to the inputs.[2] Thus, not only is estimating the inputs of the mean–variance model a challenging task, it is arguably the most critical step affecting portfolio performance. Robust portfolio models address this issue by reducing the sensitivity to variations in the input estimates.

Robust Portfolio Optimization

The most widely studied robust method is robust portfolio optimization.[3] In robust optimization, possible input values are specified in advance and the worst-case approach is employed for finding the portfolio that performs well in even the worst situation within the predefined set of values. The set of possible input values are usually referred to as the *uncertainty set*, and there is much freedom in constructing uncertainty sets. For example, uncertainty sets can be defined for mean asset returns only or for multiple input types such as means, variances, and covariances of asset returns.

Nonetheless, the selection of uncertainty sets is critical in robust portfolio optimization because it determines whether a robust formulation can be efficiently solved; robust problems will not be applicable in practice if robust allocations cannot be computed in a timely manner with standard optimization solvers.[4] One common choice for uncertainty sets in robust mean–variance portfolios is to determine a componentwise interval of possible values for every mean, variance, and covariance estimate required by the model. Thus, each component is assumed to deviate within a separate interval. It is also possible to form an uncertainty set with an elliptical shape in high-dimensional space representing possible values around a close distance to a point estimate. Regardless of the shape of uncertainty sets, robust portfolio optimization provides asset managers with worst-case estimations and, as a result, forms robust portfolios that account for worst situations.

Robust Portfolio Management in Practice

In general, the worst-case consideration of robust models is suitable for modeling worst scenarios in investments. It is mostly applicable in strategic asset allocation for gaining diversification, stability, and performance over the long horizon. Moreover, robust portfolio optimization achieves robustness through solving an optimization problem, and this becomes a great strength in practice because instabilities induced by constraints can also be formulated as robust problems. Hence, robust models can be expanded to include various uncertain situations in strategic asset allocation.

Although the implementation of worst-case optimization is based on mathematical foundations, the parameter values used in performing robust optimization are usually chosen empirically based on experience and analyses that fit each investment situation. For example, determining the level of robustness may depend on many details such as investment objective, time horizon, investment universe, and candidate assets.[5] Nonetheless, there seems to be consensus that modeling uncertainty in only expected asset returns is sufficient for achieving robust performance and that elliptical uncertainty sets for expected returns are most effective in practice.

Asset managers should be aware that high robustness may lead to conservative investments that may not be attractive under normal market conditions. In addition to adjusting the shape and size of uncertainty sets, one vendor suggests including an additional constraint that balances assets that have better-than-expected returns with assets that have worse-than-expected returns (Ceria and Stubbs [2006]). This constraint, referred to as the *zero net alpha adjustment*, assumes that not all assets will be in their worst condition simultaneously in a realistic worst scenario; on average, there will be some assets with higher values than expected and some assets with lower values than expected. The experience of asset managers supports the effectiveness of the zero net alpha adjustment in reducing excessive conservativeness of robust portfolio optimization.

MODELING UNCERTAINTY USING FACTOR MODELS

Whereas the standard approach for worst-case models constructs uncertainty sets directly for inputs of the mean–variance model at the asset level such as means, variances, and covariances of asset returns, uncertainty can be modeled at the factor level by incorporating factor models into structuring uncertainty sets. Because factors represent sources of major movements in the market, uncertainty at the factor level may be a better choice for modeling estimation errors than ambiguity at the individual asset level. This section summarizes developments in modeling uncertainty using factor models for well-known portfolio models such as mean–variance problems, value-at-risk (VaR) problems, and conditional value-at-risk (CVaR) problems.

Robust Mean–Variance Models

One of the pioneering studies on robust portfolio optimization using factor models was presented by Goldfarb and Iyengar [2003]. They develop robust portfolio formulations in which the uncertainty in asset returns is modeled from uncertainty arising from various linear factor model components, such as

$$r_i = \mu_i + \sum_{j=1}^{m} V_{j,i} f_j + \epsilon_i \quad \text{for } i = 1, \dots, n$$

or in matrix form as

$$r = \mu + V^T f + \epsilon, \tag{1}$$

where n is the number of assets, m is the number of factors (m is usually much smaller than n), r is the vector of random returns of the n assets, μ is the vector of mean returns of the n assets, f is the vector of random returns of the m factors, V is the factor loading matrix ($V_{j,i}$ is the sensitivity of the ith asset to the jth factor), and ϵ is the vector of residual returns. The covariance matrix of residual returns is denoted as D, which is assumed to be a diagonal matrix with only nonzero variances of residual returns.

In their model, uncertainty exists in three components and the uncertainty sets are defined separately:

1. Mean return vector μ (componentwise interval uncertainty set)
2. Factor-loading matrix V (elliptical uncertainty set)
3. Covariance matrix D (componentwise interval uncertainty set).

Based on these uncertainty sets, they derive robust versions of several mean–variance portfolio formulations that are efficiently solved with comparable computational complexity as the classical nonrobust portfolio problems. For the remainder of this article, we refer to the factor model given by Equation 1, along with the above uncertainty sets, 1–3, as the *robust factor model* (RFM).

RFM is a separable model because it defines separate uncertainty sets for different components of the factor model. Lu [2011a, 2011b] proposes a joint uncertainty set for the same linear factor model given by Equation 1. The joint uncertainty set defines a combined ellipsoidal set for the mean return vector μ and the factor-loading matrix V. The joint uncertainty set specifies a single value representing the model's overall robustness level; and the joint ellipsoid shape leads to improved portfolio performance because eliminating the corner cases of the separable model forms more diversified portfolios. Several robust formulations with a joint uncertainty set can be efficiently solved and thus present another viable option as a robust investment strategy.

Some studies extend the use of RFM but take different approaches for constructing robust portfolios. Ma, Zhao, and Qu [2008] derive a worst-case utility maximization problem in which they introduce a concave–convex utility function measuring the utility of investors under uncertainty. The concave–convex function represents concave utility for gains (diminishing marginal utility with gain) and convex utility for losses (opposite behavior to gains). They show that the robust utility maximization problem can be solved efficiently.

Li and Kwon [2013] propose an approach to incorporate tail events into robust formulations. Because expanding the uncertainty set to include extreme events will also elevate its conservativeness, they divide scenarios into groups that fall inside and outside of predefined regions. Thus, they separately consider scenarios that are relatively more extreme and form robust portfolios when uncertainty exists in the mean vector and covariance matrix of factor returns.

Robust Portfolio Models with Various Risk Measures

Concerns over representing investment risk with variance in the classical mean–variance model have motivated alternative measures of portfolio risk. Similarly, factor-based robust formulations with various risk measures intend to resolve limitations in the use of variance. In the study by Goldfarb and Iyengar [2003], one of the robust formulations presented is a robust VaR portfolio problem. They show that the robust VaR problem can also be efficiently solved when representing uncertainty in asset returns with their robust factor model.

Robust VaR optimization is also addressed by El Ghaoui, Oks, and Oustry [2003]. The main focus of their study is on formulating the worst-case VaR problem when the true distribution of returns is only partially known, such as the mean and covariance matrix. Optimal robust allocations can be easily computed when the mean vector and the covariance matrix of factor returns are assumed to be within componentwise intervals and when uncertainty in the residual return variances are defined by componentwise bounds. Alternatively, focusing on the uncertainty in factor loadings finds an upper bound on the worst-case VaR. These three types of uncertainty sets model the source of ambiguity in different ways, but all formulations are solved efficiently and provide valuable information on robust investing.

Instead of using the mean and covariance matrix of factor returns, Natarajan, Pachamanova, and Sim [2008] allow asymmetric distributions in their model. They derive a robust VaR model for an uncertainty set on factor returns that relies on asymmetric variability measures, which they refer to as asymmetry-robust VaR (ARVaR) problem for a factor model of returns. Their formulation generalizes the robust VaR model of El Ghaoui, Oks, and Oustry [2003], and it adds importance in portfolio risk management because ARVaR defines a coherent risk measure. The advantage of asymmetry information is confirmed with small-cap stocks of the S&P 600 Index because these stocks tend to have more skewed return distributions compared to large-cap stocks.

There are also cases in which RFM is implemented in robust CVaR problems. Based on the worst-case CVaR formulation of Zhu and Fukushima [2009], Ruan and Fukushima [2012] develop worst-case CVaR that utilizes factor models for modeling uncertainty in returns. Instead of computing CVaR from a known probability distribution, they consider a set of distributions that result in robust portfolios. More specifically, under the assumption of multivariate normal distribution with a fixed covariance matrix, the set of possible mean vectors is determined from a factor model. In their analysis, uncertainty sets constructed with the Fama–French three-factor model from returns during market downturns are shown to provide safe investments, especially during market downturns.

Uncertainty in Factor Models

As discussed, many studies use RFM or a similar linear factor model to describe the source of estimation error in forming investment decisions. Nonetheless, the choice of factors is unrestricted and often left for portfolio managers to decide. Thus, managers are naturally led to wondering if a particular factor model is ideal under a given situation. The possibility of relying on a suboptimal factor model or the existence of multiple candidate factor models introduces another view on model uncertainty and several models are introduced here.

Garlappi, Uppal, and Wang [2007] incorporate two aspects of uncertainty: parameter uncertainty and model uncertainty. They consider a situation in which a manager relies on a factor model for generating asset return estimates, and the manager is not only uncertain about the asset returns but also lacks confidence in the particular factor model. The formulation includes two ellipsoidal uncertainty sets for the returns of assets and factors, respectively. They demonstrate the effect of each type of uncertainty on portfolio composition and the importance of the relative level of uncertainty between the two in determining the optimal robust allocation. These findings are also empirically confirmed in the simplest case in which the capital asset pricing model (CAPM) is chosen as the return-generating model.

Lutgens and Schotman [2010] apply the max–min approach for finding robust portfolios when there are multiple experts providing multiple estimates.

Each expert is assumed to use a linear factor model for estimating values that may differ from other experts. Lutgens and Schotman discuss the composition of optimal robust portfolios when the experts' underlying factor models are either similar or conflicting. The authors also demonstrate robust performance through empirically testing robust portfolios constructed with experts' views from the CAPM and the Fama–French three-factor model. Two cases, uncertainty only in expected returns and uncertainty in both expected returns and covariance matrix, are considered.

Even though multiple candidates for factor models are not explicitly addressed, Glasserman and Xu [2013] study ambiguity that arises from the evolvement of the underlying factors of market movements. Hence, in their setting, factor models as well as factors themselves evolve stochastically and the degree of uncertainty is limited using relative entropy. The authors formulate finite and infinite horizon problems that take transaction costs into account, and they demonstrate the out-of-sample robustness with commodity portfolios in which factors are chosen as moving averages of futures prices.

Factor-Based Models in Practice

Factor models play several roles in robust investment strategies. One approach is to perform factor model analyses for acquiring factor-based estimates of asset returns, as well as market movements, prior to the allocation step. Thus, factor models result in meaningful inputs for robust models, and robust models result in robust allocations. Another approach is to formulate robust portfolio problems in which the optimal weights are allocations in factors. These strategies produce robust portfolios investing in factor indices or portfolios with robust exposure in desired factors. In terms of the types of factor models that are popular, macroeconomic, fundamental, and statistical factor models are all considered and the factors are chosen independently for each investment strategy depending on the investment characteristics.

ROBUST FACTOR-BASED STRATEGIES

In this section, we further discuss investment strategies based on robust factor-based models. We begin by examining passive strategies for tracking an index and then introduce techniques for gaining additional alpha in robust strategies.

Robust Index Tracking

Index-tracking strategies have specific goals of closely following a given index; the primary objective is not to beat the index but to match index returns. Thus, robustness is arguably more important than in active strategies when full replication is impractical and a subset of index constituents must be chosen.

Erdoğan, Goldfarb, and Iyengar [2004] propose a robust maximum Sharpe ratio problem for tracking capitalization-weighted broad market indices. Motivated by the CAPM and the Black–Litterman model [1991], their strategy aims to find the portfolio with maximum Sharpe ratio, which represents the market portfolio of the CAPM, and computes the expected return vector from equilibrium risk premiums used in the Black–Litterman model (He and Litterman [1999]). Erdoğan, Goldfarb, and Iyengar [2004] find that modeling the uncertainty in expected returns does not improve the performance of their model and thus construct uncertainty sets for the factor-loading matrix and the covariance matrix of residual returns of RFM. By adding a constraint for controlling portfolio beta and including weight adjustments for reducing transaction costs, their max–min formulation is demonstrated to be an effective index-tracking strategy with fewer stocks. In their experiments, they select eigenvectors of the return covariance matrix and six major market indices as the set of factors. Moreover, they also comment on the advantage of using their robust formulation as a passive investment strategy through a buy-and-hold approach with their robust index-tracking portfolios.

Another robust index-tracking problem is formulated by Kwon and Wu [2016]. Their formulation is more straightforward for forming index-tracking portfolios: the objective is to maximize expected return of a tracking portfolio subject to a limit on its maximum tracking error. They model the uncertainty in returns through RFM and their factor-based robust strategy is illustrated for tracking the S&P 100 Index with the Fama–French three-factor model.

Robust Strategies for Improving Performance

Because residual returns not explained by a given benchmark may generate alpha returns, active investment strategies can exploit the advantage by modeling the ambiguity in residual returns. Erdoğan, Goldfarb, and Iyengar [2004] model residual returns using RFM, and the robust formulation for maximizing the information ratio with adjustments for reducing transaction costs is shown to have superior performance when the S&P 500 Index is set as the benchmark. Their robust active investment approach is further studied by Erdoğan, Goldfarb, and Iyengar [2006]. Specifically, they provide an example of modeling transaction costs and reformulate it into an efficient optimization problem. They also show how to limit average deviation in alpha, incorporate investors' views, and implement a data-driven approach to expand its use in practice.

Asl and Etula [2012] share a practical factor-based approach to strategic asset allocation. Their six-factor model captures distinct sources of long-term returns across different asset classes: equity (market risk), term (inflation and interest rate risk), funding (risk in short-term credit conditions), liquidity (risk in marketwide liquidity conditions), FX (systematic exchange rate risk), and EM (risks specific to emerging markets). They demonstrate the strength of robust factor-based investing by applying robust portfolio optimization to asset allocation where their six-factor model is used for estimating expected returns of various asset classes. The robust factor-based approach is empirically shown to improve robustness as well as efficiency within the mean–variance framework.

Although differing from the worst-case approach of many robust models, the Saxena and Stubbs [2013] approach derives a variation of the general multifactor model, as given by Equation 1, for improving risk estimation that eventually results in robust performance. They argue that most factor models underestimate the risk of systematic risk factors that are not included in the particular factor model, and this causes underestimation of the optimized portfolio risk. As a solution, Saxena and Stubbs recommend considering the orthogonal component as an additional factor, which they refer to as the *alpha alignment factor*. Using this factor can reduce the inconsistency between the estimated and realized portfolio risk.

Cheung [2013] modifies the Black–Litterman model [1991], which is widely accepted as an effective approach for improving portfolio robustness, to incorporate the investor's view on factor movements. Whereas the original Black–Litterman model allows investors to insert views on asset returns, the proposed augmented version supports personal views on factor returns and idiosyncratic returns of assets, in addition to asset returns. Cheung [2013] provides a few specific examples on posterior returns; the investment universe is set to the constituents of the FTSE Eurotop 100 Index, and the relevant factors include industries and various fundamental factors.

Practitioners' Insights on Robust Factor Investing

For factor-based investing in general, one vendor stresses the value of incorporating the alpha alignment factor (Saxena and Stubbs [2013]). Adjusting the misalignment between expected returns and risk models reveals significant improvements in practice. One portfolio manager also mentions that managers use the alpha alignment factor with robust optimization in fundamental and statistical models.

Unfortunately, although practitioners believe that robust optimization is also applicable in recent factor-based smart beta strategies, there does not seem to be a noticeable attempt to apply it in the asset management industry. However, there is consensus that the current status does not reflect the limitation of robust models, and the value of worst-case approaches have ample possibilities in portfolio management, including factor-based investment strategies.

FACTOR EXPOSURE OF ROBUST PORTFOLIOS

In this section, we summarize several studies that reveal the increased factor exposure of portfolios constructed from robust optimization. Although these studies do not directly offer robust investment strategies, their findings suggest applying robust methods with caution, especially when it is important to control a portfolio's factor exposure for risk management.

With a focus on robust portfolio formulations with interval or ellipsoidal uncertainty sets on expected asset returns, Kim et al. [2013] examine the relationship between robust portfolio returns and the returns of the Fama–French three factors. They show empirically that robust portfolios have higher factor dependency than classical mean–variance portfolios under various conditions. Furthermore, robust portfolios show higher factor dependency as the robustness level is increased. Kim, Kim, and Fabozzi [2014a] arrive at similar conclusions. They first show analytically that robust portfolio weights move closer to the portfolio with the highest factor dependency, and then illustrate through simulation and historical market data that increasing robustness shifts portfolio weights toward the portfolio that is maximally explained by factor returns.

Although this may be a concern when performing robust optimization to improve investment performance robustness, Kim et al. [2014] develop several robust formulations that control the factor tilting of robust optimization. By limiting the shift toward increased factor dependency with linear constraints, the proposed formulations produce robust portfolios with the desired factor exposures without affecting computational complexity.

CONCLUSION

Factor models are essential components for constructing portfolios—and understanding factors of various markets is critical for managing investment risk. Even though factor models provide portfolio managers with marketwide insight as well as individual asset information, ambiguity may exist in the evolution of factors or the relationship between factors and assets. In other words, incorporating parameter uncertainty or model uncertainty of factor models can improve various investment strategies and analyses that rely on factor models.

In this article, we review studies on robust approaches for factor-based investing. A popular topic is the use of robust factor models in robust portfolio optimization. These models formulate a worst-case problem in which the possible situations are expressed with a multifactor model. The models summarized are applicable to asset allocation, portfolio selection, passive strategies such as index tracking, and active strategies. Moreover, experience from portfolio managers provides further insight on the value of robust investing.

ENDNOTES

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[1]Insight on robust factor-based investing in practice is based on communication with Robert A. Stubbs of Axioma and Jakub Duda of Goldman Sachs Private Wealth Management. In particular, we present the use of robust optimization by managers with a focus on relation to factor models.

[2]See, for example, Michaud [1989], Best and Grauer [1991], and Chopra and Ziemba [1993].

[3]An overview on robust portfolio optimization is presented in Fabozzi et al. [2007] and Kim, Kim, and Fabozzi [2015].

[4]Some examples on uncertainty sets for mean–variance portfolios are introduced in Lobo and Boyd [2000] and Kim, Kim, and Fabozzi [2014b].

[5]An increase in the level of robustness can be implemented by defining a larger uncertainty set that includes more extreme values.

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